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FIRST MOTIONS FROM NONUNIFORMLY MOVING DISLOCATIONS

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Abstract—The response of an elastic half space to a realistic model of faulting is considered. A dislocation is assumed to be developed along a line of finite length and then moves nonuniformly along an inclined plane (fault) surface. Analytical solutions to the surface displacement in the form of double integrals are derived by Cagniard De-Hoop technique. Nature of wave arrivals at the surface are discussed both in case of a decelerating and an accelerating source. First motion responses are obtained near different wave arrivals by a limiting process.

1. INTRODUCTION

Although exact solutions for the response of an elastic half space to uniformly moving sources are known for quite some time (see for current bibliography, Gakenheimer and Miklowitz[1]) only recently Freund[2, 3] discussed the wave motions expected in case of a non-uniformly expanding line load. Beitin[4], Blowers[5] and Roy[6] obtained the displacement field in an elastic half space for some special types of nonuniform motion. For accoustic case the problem has been considered by Aggarwal and Ablow[7] and Stronge[8].

Currently there has been increasing interest in theoretical elastic motions near a fault in view of possible applications in the design of earthquake resistant structures in close proximity to faults. Two dimensional kinematic fault models have been used by Boore and Zoback[9, 10] and Niazy[11] etc. to explain the strong motion recordings of earthquakes. Radiation pattern of uniformly propagating faults has been considered earlier by Knopoff and Gilbert[12] and Savage[13]. Near field simulation method using numerical integration of a point source dislocation following Haskell[14] has been used by others[15, 16] to explain the strong motion accelerograph data. Theoretical models of seismic source mechanism viz. a distribution of single or double couples or dislocation over a finite area and moving with constant velocity has long been used[17] to estimate the parameters of a fault, e.g. velocity and length, by comparing theoretical amplitude spectra with those of actual seismographs. Another line of attack is to assume earthquake source models as dynamic cracks[18, 19].

In all such works mentioned above, the source expands with uniform velocity. However the motion at the focus of a shallow earthquake is more likely to be nonuniform because of heterogeneity near the focus. Thus a nonuniformly moving source is a more accurate representation of the mechanism of an earthquake than a uniformly moving source. Hence we represent the seismic source by means of dislocation over a finite length which then moves nonuniformly perpendicular to its initial position along an inclined plane fault surface. Analytical solutions to the displacement on the surface of an elastic half space are obtained in the form of double integral over finite ranges. It is observed that theoretical seismograms are expected to differ considerably from those of a stationary or uniformly moving sources.

2. MATHEMATICAL FORMULATION

We consider a homogeneous elastic half space and take the earthquake source to involve a plane (fault) surface across which displacement discontinuity suddenly arises along a line of finite length which then moves perpendicular to itself along the fault surface. We introduce a right handed coordinate system $(\zeta_1, \zeta_2, \zeta_3)$ (Fig. 1) to describe the source system such that $\zeta_3 = 0$ represents the fault plane with ζ_1 taken along the direction of motion. The other coordinate system (x, y, z) is chosen such that $z = -h$ represents the free bounding surface of the elastic

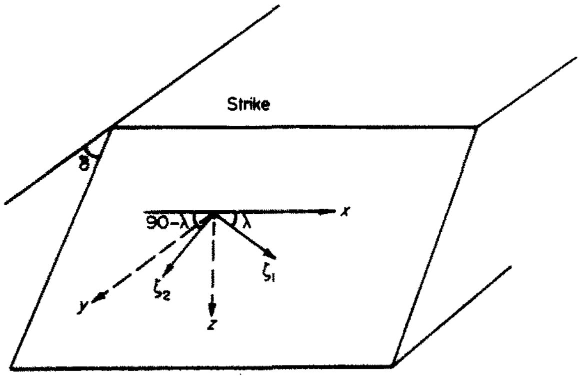

Fig. 1. Coordinate system.

half space and x-axis is taken along the strike of the fault plane. The transformation rule for the co-ordinate systems

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{pmatrix} \tag{1}$$

where $A = [a_{ij}]$ is the (3×3) transformation matrix.

In terms of the slip angle λ and dip angle δ we have

$$A = \begin{pmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda\cos\delta & \cos\lambda\cos\delta & -\sin\delta \\ \sin\lambda\sin\delta & \cos\lambda\sin\delta & \cos\delta \end{pmatrix}. \tag{2}$$

Let (u_1, u_2, u_3) and (u, v, w) be the components of the displacement $\mathbf{u}$ along $(\zeta_1, \zeta_2, \zeta_3)$ and (x, y, z) respectively. Let the earthquake source be represented by a prescribed tangential displacement discontinuity across the fault plane $\zeta_3 = 0$. We take

$$[u_1] = H(\zeta_1 - S(t))[H(\zeta_2) - H(\zeta_2 - b)][H(t) - H(t - t_1)] \tag{3}$$

where $[u_1]$ is the jump in the ζ_1-component of displacement across $\zeta_3 = 0$, $H(t)$ is Heaviside unit function and b is the extent along ζ_2-axis.

For our purpose it is sufficient to consider the dislocation given by

$$[u_1] = H(\zeta_1 - S(t))H(\zeta_2)H(t). \tag{4}$$

Then the displacement field in case of dislocation as given by (3) can be written from those of (4). We note that no restrictions need be imposed on $S(t)$ except that it is monotone and $S(0) = 0$.

The body force equivalents $\mathbf{F}$ with components (F_1, F_2, F_3) in the $(\zeta_1, \zeta_2, \zeta_3)$ coordinate system for the dislocation given by (4) can be written as (Burridge and Knopoff[12]).

$$F_1 = -\mu H(\zeta_1 - S(t))H(\zeta_2)\delta'(\zeta_3)H(t), \qquad F_2 = 0,$$
$$F_3 = -\mu\delta(\zeta_1 - S(t))H(\zeta_2)\delta(\zeta_3)H(t). \tag{5}$$

Roy[20] has given a general method of finding the surface displacements in an elastic half space associated with arbitrary force system $\mathbf{F}$. The use of two different cartesian coordinate systems, one of which describes the source condition and the other is chosen such that it is perpendicular to the bounding surface, is found to be specially suitable for obtaining the displacement field in case of dislocation across an inclined fault surface. Following[20] the transformed displacement at the surface $z = -h$ can be written as

$$\bar{\mathbf{u}}(-h) = \bar{\mathbf{u}}_P(-h) + \bar{\mathbf{u}}_S(-h) + \bar{\mathbf{u}}_{SH}(-h), \tag{6}$$

where

$$\bar{\mathbf{u}}(-h)=\int_0^\infty \mathbf{u}(-h)\,e^{-pt}\,dt, \qquad \mathbf{u}(-h)=\mathbf{u}(x,y,-h,t),$$

$$\bar{\mathbf{u}}_P(-h)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{A}{4\pi^2\mu}\frac{e^{i(\xi x+\eta y)-\zeta_P h}}{F(\xi,\eta)}\left[-2\zeta_S(\xi\mathbf{e}_x+\eta\mathbf{e}_y)+i\left(2\xi^2+2\eta^2+\frac{p^2}{\beta^2}\right)\mathbf{e}_z\right]d\xi\,d\eta,$$

$$\bar{\mathbf{u}}_S(-h)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{B\,e^{i(\xi x+\eta y)-\zeta_S h}}{4\pi^2\mu F(\xi,\eta)}[(2\xi^2+2\eta^2+p^2/\beta^2)(\xi\mathbf{e}_x+\eta\mathbf{e}_y)-2i(\xi^2+\eta^2)\zeta_P\mathbf{e}_z]\,d\xi\,d\eta,$$

$$\bar{\mathbf{u}}_{SH}(-h)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{C(\eta\mathbf{e}_x-\xi\mathbf{e}_y)}{4\pi^2\mu(\xi^2+\eta^2)\zeta_S}\,e^{i(\xi x+\eta y)-\zeta_S h}\,d\xi\,d\eta, \tag{7}$$

where $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are unit vectors along x, y and z axes respectively, and

$$A=D_P\xi+E_P\eta-iG_P\zeta_P, \qquad B=(D_S\xi+E_S\eta)\frac{\zeta_S}{(\xi^2+\eta^2)}-iG_S,$$

$$C=D_S\eta-E_S\xi, \qquad D=\bar{F}_1^{kvs}a_{11}+\bar{F}_2^{kvs}a_{12}+\bar{F}_3^{kvs}a_{13},$$

$$E=\bar{F}_1^{kvs}a_{21}+\bar{F}_2^{kvs}a_{22}+\bar{F}_3^{kvs}a_{23}, \qquad G=\bar{F}_1^{kvs}a_{31}+\bar{F}_2^{kvs}a_{32}+\bar{F}_3^{kvs}a_{33},$$

$$\zeta_P=(\xi^2+\eta^2+p^2/\alpha^2)^{1/2},\ (Re(\zeta_P)>0), \qquad \zeta_S=(\xi^2+\eta^2+p^2/\beta^2)^{1/2},\ (Re(\zeta_S)>0),$$

$$\alpha^2=(\lambda+2\mu)/\sigma, \qquad \beta^2=\mu/\sigma,$$

$$F(\xi,\eta)=(2\xi^2+2\eta^2+p^2/\beta^2)^2-4(\xi^2+\eta^2)\zeta_P\zeta_S,$$

$$(\bar{F}_1^{kvs},\bar{F}_2^{kvs},\bar{F}_3^{kvs})=\int_0^\infty\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(F_1,F_2,F_3)\,e^{-i(k\zeta_1+v\zeta_2+s\zeta_3)-pt}\,d\zeta_1\,d\zeta_2\,d\zeta_3\,dt. \tag{8}$$

λ, μ and σ are Lame's constants and density of the medium. D_P, D_S etc. are obtained from D etc. after first changing over from variables (k, v, s) to (ξ, η, ζ) through the relation

$$\begin{pmatrix}k\\v\\s\end{pmatrix}=A^{-1}\begin{pmatrix}\xi\\\eta\\\zeta\end{pmatrix}=\begin{pmatrix}\cos\lambda & \sin\lambda\cos\delta & \sin\lambda\sin\delta\\-\sin\lambda & \cos\lambda\cos\delta & \cos\lambda\sin\delta\\0 & -\sin\delta & \cos\delta\end{pmatrix}\begin{pmatrix}\xi\\\eta\\\zeta\end{pmatrix} \tag{9}$$

and setting $\zeta=-i\zeta_P$ and $\zeta=-i\zeta_S$ respectively.

The transformed surface displacement for the given dislocation can be obtained from (6)–(8) after obtaining the values of D_P, D_S etc. for the body force equivalent as given by (5). They can be shown to be of the form

$$\bar{N}_P=\int_0^\infty\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{P(\xi,\eta,p)}{k_P\nu_P}\,e^{i(\xi X+\eta Y)-Z(\xi^2+\eta^2+p^2/\alpha^2)^{1/2}-p\tau}\,d\xi\,d\eta\,d\tau, \tag{10a}$$

$$\bar{N}_S=\int_0^\infty\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{Q(\xi,\eta,p)}{k_S\nu_S}\,e^{i(\xi X+\eta Y)-Z(\xi^2+\eta^2+p^2/\beta^2)^{1/2}-p\tau}\,d\xi\,d\eta\,d\tau, \tag{10b}$$

where

$$X=x-S(\tau)\cos\lambda, \qquad Y=y-S(\tau)\sin\lambda\cos\delta,$$
$$Z=h+S(\tau)\sin\lambda\sin\delta. \tag{10c}$$

Complete expressions for $P(\xi,\eta,p)$, $Q(\xi,\eta,p)$, k_P etc. are given in Appendix 1. The terms k_P, k_S etc. in the denominator arise due to $H(\zeta_2)$ term in (5). The apparent singularity at $k_P=0$, $k_S=0$ etc. could be avoided if we use Heisenberg's delta function[21].

3. INVERSION

Evaluation of the integral of the type (10a) and (10b) will be done by Cagniard De-Hoop technique. Following the method of Gekenheimer and Miklowitz[1] we make the De-Hoop transformation

$$\xi = \frac{\rho}{\alpha}(q \cos \phi - w \sin \phi), \qquad \eta = \frac{\rho}{\alpha}(q \sin \phi + w \cos \phi), \tag{11}$$

where

$$\tan \phi = \frac{R}{Z}, \qquad R = (X^2 + Y^2)^{1/2}.$$

The integration path which is along the real q-axis is deformed to Cagniard's path, viz.

$$\begin{aligned} \alpha t &= \alpha \tau - iqR + Z(q^2 + w^2 + 1)^{1/2}, \\ \alpha t &= \alpha \tau - iqR + Z(q^2 + w^2 + \alpha^2/\beta^2)^{1/2}, \end{aligned} \tag{12}$$

depending on whether we consider $\bar{N}_P$ or $\bar{N}_S$.

We note that in transforming from the real q-axis to the Cagniard's path in the q-plane, as remarked earlier, $k_P = 0$, $\nu_P = 0$ etc. in the denominator of (10a) and (10b) are not poles. On performing the inversion, the surface displacement can be written as (see, for detail, Roy[22] in a similar case)

$$\mathbf{u}(-h) = \mathbf{u}_P(-h) + \mathbf{u}_S(-h) + \mathbf{u}_{SH}(-h) + \mathbf{u}_{SP}(-h), \tag{13}$$

where

$$\mathbf{u}_j(-h) = \int_0^t H(t - \tau - \rho/V_j)\mathbf{I}_j \, d\tau, \tag{14}$$

$$\rho = (X^2 + Y^2 + Z^2)^{1/2},$$

$$\mathbf{I}_j = \frac{\alpha}{2\pi^2 \rho} Re \int_{-\pi/2}^{\pi/2} \mathbf{M}_j(q_j, \omega_j)(q_j^2 + \omega_j^2 + d_j^2)^{1/2} \, d\psi \tag{15}$$

j has the values P, S or SH;

$$\left.\begin{aligned} d_j &= 1 && \text{for } j = P \\ &= \alpha/\beta && \text{for } j = S \text{ or } SH \end{aligned}\right\}$$

and

$$V_P = \alpha, \qquad V_S = V_{SH} = \beta.$$

Also

$$\begin{aligned} \mathbf{u}_{SP}(-h) = {} & \int_0^t \left[H\left(t - \tau - \frac{R}{\alpha} - Z\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}\right) - H\left(t - \tau - \frac{\rho}{\beta}\right)\right] H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right)\mathbf{I}_{SP1} \, d\tau \\ & + \int_0^t \left[H\left(t - \tau - \frac{\rho}{\beta}\right) - H\left(t - \tau - \frac{\rho^2}{Z}\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}\right)\right] H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right)\mathbf{I}_{SP2} \, d\tau, \end{aligned} \tag{16}$$

where

$$\mathbf{I}_{SP1} = -\frac{\alpha}{\pi^2 \rho} Re \int_{-\pi/2}^{\pi/2} \frac{i(\omega_{SP1}^2 + q_{SP1}^2 + \alpha^2/\beta^2)^{1/2}}{(\chi^2 + \sin^2 \psi)^{1/2}} \mathbf{M}_S(q_{SP1}, \omega_{SP1}) \cos \psi \, d\psi, \tag{17}$$

and

$$\mathbf{I}_{SP2} = -\frac{\alpha}{\pi^2 \rho} Re \int_{-\pi/2}^{\pi/2} \frac{i\delta' \mathbf{M}_S(q_{SP2}, \omega_{SP2})(q_{SP2}^2 + \omega_{SP2}^2 + \alpha^2/\beta^2)^{1/2} \cos \psi \, d\psi}{\left[\delta'^2 \sin^2 \psi + \frac{R^2}{\rho^2}\left(\frac{\beta^2 (t - \tau)^2}{\rho^2} - 1\right)\right]^{1/2}}. \tag{18}$$

Meaning of the different symbols used in (15)–(17) are given in Appendix 1.

4. FURTHER REDUCTION OF THE SURFACE DISPLACEMENT

The surface displacements as given by (14) are written in alternate forms which are suitable for further reduction. Thus

$$\begin{aligned}\mathbf{u}_j(-h) = \int_0^{t-(\zeta_2^2+\zeta_3^2)^{1/2}/V_j} & [H(\zeta_1 - S(\tau) + [V_j^2(t-\tau)^2 - \zeta_2^2 - \zeta_3^2]^{1/2}) \\ & - H(\zeta_1 - S(\tau) - [V_j^2(t-\tau)^2 - \zeta_2^2 - \zeta_3^2]^{1/2})]\mathbf{I}_j \, d\tau, \end{aligned} \tag{19}$$

where ζ_1, ζ_2, ζ_3 are obtained from inverting (1) and setting $z = -h$. Thus

$$\begin{aligned} \zeta_1 &= x \cos\lambda + y \sin\lambda \cos\delta - h \sin\lambda \sin\delta, \\ \zeta_2 &= -x \sin\lambda + y \cos\lambda \cos\delta - h \cos\lambda \sin\delta, \\ \zeta_3 &= y \sin\delta + h \cos\delta. \end{aligned} \tag{20}$$

Note that from now on, unless otherwise stated, $(\zeta_1, \zeta_2, \zeta_3)$ refer to points on surface $z = -h$. The region of support for the τ-integration is bounded by the curve

(I) $$\zeta_1 = S(\tau) - [V_j^2(t-\tau)^2 - \zeta_2^2 - \zeta_3^2]^{1/2}, \tag{21}$$

(II) $$\zeta_1 = S(\tau) + [V_j^2(t-\tau)^2 - \zeta_2^2 - \zeta_3^2]^{1/2}. \tag{22}$$

Figures 2(a–d) show the curves I and II as τ varies from 0 to $(t - (\zeta_2^2 + \zeta_3^2)^{1/2}/V_j)$. The curve I is a monotonic increasing function as τ varies from 0 to $(t - (\zeta_2^2 + \zeta_3^2)^{1/2}/V_j)$. Curve II has extremum at points where

$$\frac{\partial \zeta_1}{\partial \tau} = \dot{S}(\tau) - \frac{V_j^2(t-\tau)}{[V_j^2(t-\tau)^2 - \zeta_2^2 - \zeta_3^2]^{1/2}} \tag{23}$$

vanish. Also we have

$$\frac{\partial^2 \zeta_1}{\partial \tau^2} = \ddot{S}(\tau) - \frac{\zeta_2^2 + \zeta_3^2}{[V_j^2(t-\tau)^2 - \zeta_2^2 - \zeta_3^2]^{3/2}}. \tag{24}$$

Depending on the nature of the moving source, the curve II may have a single maximum at $\zeta_1 = \zeta_{j1}^{\dagger}$ (or equivalently at $\tau = \tau_j^{\dagger}$ or $t = t_j^{\dagger}$) or both a maximum at $\zeta_1 = \zeta_{j1}^{\dagger}$ and a minimum at $\zeta_1 = \zeta_{j1}^{\dagger *}$ (i.e. at $\tau = \tau_j^{\dagger *}$ or $t = t_j^{\dagger *}$). We consider an accelerating source, i.e. $\ddot{S}(\tau) > 0$. In this case it is easily seen from (23) if $(\partial\zeta_1/\partial\tau)_0 > 0$ i.e. if the initial velocity $\dot{S}(0)$ (in general greater than V_j) is such that $(\partial\zeta_1/\partial\tau)_0 > 0$, then the curve II has a maximum at $\zeta_1 = \zeta_{j1}^{\dagger}$ $(\tau = \tau_j^{\dagger})$ (Fig. 2a). While if $(\partial\zeta_1/\partial\tau)_0 < 0$ i.e. if the initial velocity $\dot{S}(0)$ (in general less than V_j) is such that $(\partial\zeta_1/\partial\tau)_0 < 0$, the curve II has either no extremum as in Fig. 2(b) or both a maximum at $\zeta_1 = \zeta_{j1}^{\dagger}$ and a minimum at $\zeta_1 = \zeta_{j1}^{\dagger *}$ (i.e. at $\tau = \tau_j^{\dagger *}$ or $t = t_j^{\dagger *}$) (Fig. 2c, d). In the latter case the acceleration of the source must be rapid enough such that $\partial^2\zeta_1/\partial\tau^2$ vanishes at least once in $0 < \tau < t - (\zeta_2^2 + \zeta_3^2)^{1/2}/V_j$. While for a decelerating source (i.e. $\ddot{S}(\tau) < 0$) the curve II has a maximum at $\zeta_1 = \zeta_{j1}^{\dagger}$ if $(\partial\zeta_1/\partial\tau)_0 > 0$ and no extremum at all if $(\partial\zeta_1/\partial\tau)_0 < 0$.

From the above discussion and from Fig. 2(a–d) it follows that τ as given by (22) is multiple valued in $\zeta_{j1}^{\dagger *} < \zeta_1 < \zeta_{j1}^{\dagger}$, when both $\zeta_{j1}^{\dagger}$ and $\zeta_{j1}^{\dagger *}$ exist. The segments of the curve II are then denoted by $\tau = \tau_{j2}$ and $\tau = \tau_{j3}$ $(< \tau_{j2})$ for ζ_1 in $(\zeta_{j1}^{\dagger *}, \zeta_{j1}^{\dagger})$ and simply by $\tau = \tau_{j2}$ for values of ζ_1 in $(\zeta_{j1}^{\dagger}, \zeta_{j1}^{0})$ if only one maximum viz. $\zeta_{j1}^{\dagger}$ exists. The rest of the segments of the curves I and II, where τ is single valued, is denoted by $\tau = \tau_{j1}$.

In case of head waves (i.e. $\mathbf{u}_{SP}$) the nature of surface displacement depends critically on the existence of the maximum $x = x_{SP}^{\dagger}$ and minimum $x = x_{SP}^{\dagger *}$ of the upper curve of

$$x = S(\tau)\cos\lambda \pm \left[\left\{\alpha(t-\tau) - (h + S(\tau)\sin\lambda\sin\delta)\left(\frac{\alpha^2}{\beta^2} - 1\right)^{1/2}\right\}^2 - (y - S(\tau)\sin\lambda\cos\delta)^2\right]^{1/2}. \tag{25}$$

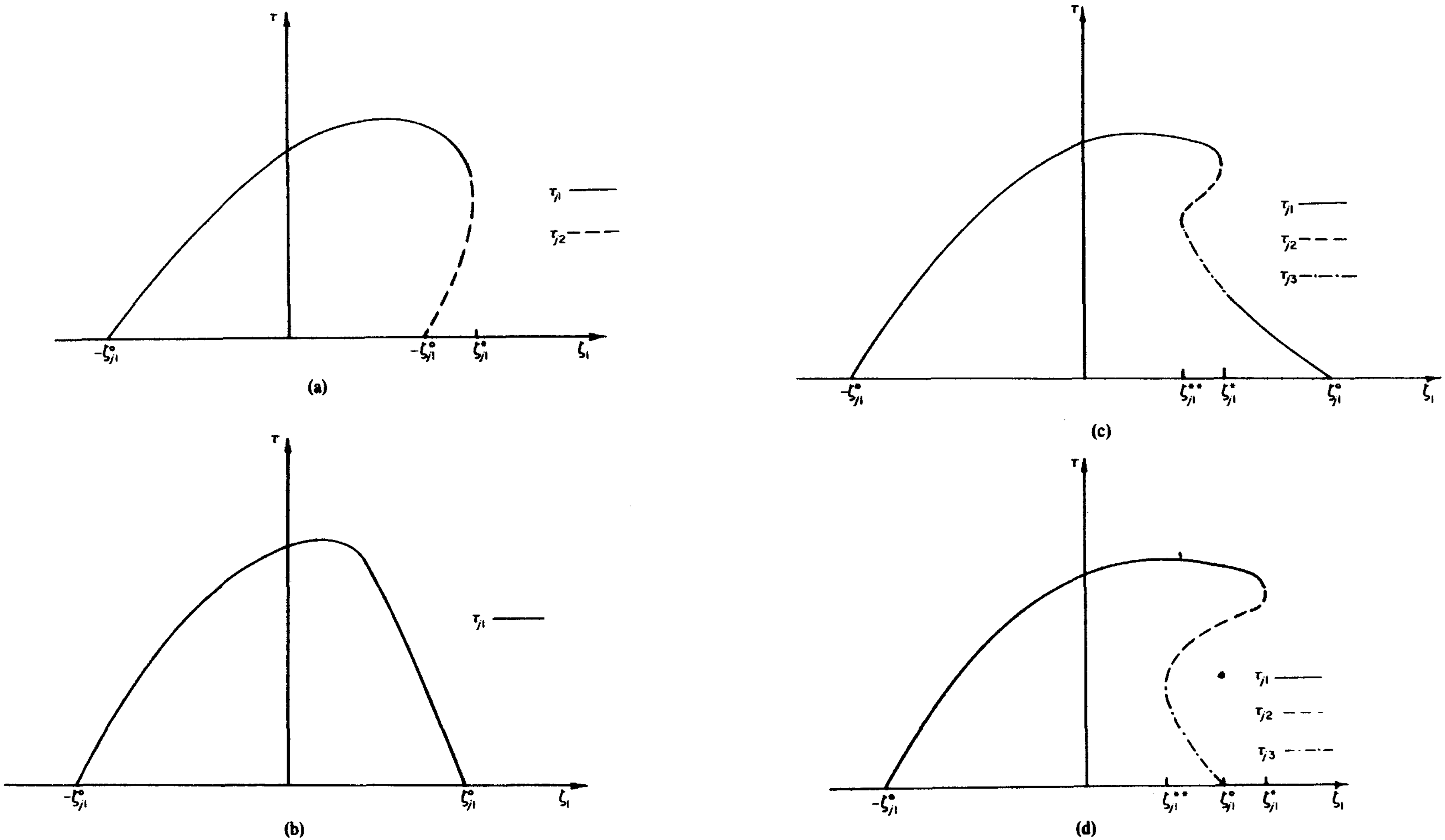

Fig. 2. Region of support of $\mathbf{I}_j$. (a) A single maximum ζ_{j1}^{*} exists, i.e. in case of a source with initial velocity such that $\dot{S}(0) > V_j$; (b) No extremum exists; (c, d) Both a maximum ζ_{j1}^{*} and a minimum ζ_{j1}^{**} exist i.e. in case of an accelerating source such that $\dot{S}(0) < V_j$ and acceleration is rapid enough.

Similarly $\zeta_1 = \zeta_{B1}^*$ and $\zeta_1 = \zeta_{B1}^{**}$ are the maximum and minimum, when they exist, of the upper curve of

$$\zeta_1 = S(\tau) \pm \left[\frac{(t-\tau)(h + S(\tau)\sin\lambda\sin\delta)}{\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}} - \zeta_2^2 - \zeta_3^2 \right]^{1/2}. \tag{26}$$

The division of the curves (25) and (26) into different segments ($\tau = \tau_{SP1}$, $\tau = \tau_{SP2}$, $\tau = \tau_{SP3}$) and ($\tau = \tau_{B1}$, $\tau = \tau_{B2}$, $\tau = \tau_{B3}$) is done exactly similar to the curves (21) and (22).

On close examination of Figs. 2(a–d) and similar figures connected with the head wave we can write the surface displacement as

$$\mathbf{u}(-h) = \mathbf{u}_P(-h) + \mathbf{u}_S(-h) + \mathbf{u}_{SH}(-h) + \mathbf{u}_{SP}(-h) \tag{27}$$

where, for $j = P$, S or SH

$$\begin{aligned}
\mathbf{u}_j(-h) = {} & [H(\zeta_1 + \zeta_{j1}^0) - G(\zeta_1 - \max^m(\zeta_{j1}^*, \zeta_{j1}^0))] \int_0^{\tau_{j1}} H\left(t - \tau - \frac{\rho}{V_j}\right) \mathbf{I}_j \, d\tau \\
& - [G(\zeta_1 - \max^m(\zeta_{j1}^*, \zeta_{j1}^0)) - H(\zeta_1 - \zeta_{j1}^0)] \int_0^{\tau_{j2}} H\left(t - \tau - \frac{\rho}{V_j}\right) \mathbf{I}_j \, d\tau \\
& - [G(\zeta_1 - \min^m(\zeta_{j1}^{**}, \zeta_{j1}^0)) - G(\zeta_1 - \min^m(\zeta_{j1}^*, \zeta_{j1}^0))] \int_{\tau_{j3}}^{\tau_{j2}} H\left(t - \tau - \frac{\rho}{V_j}\right) \mathbf{I}_j \, d\tau,
\end{aligned} \tag{28}$$

and

$$\begin{aligned}
\mathbf{u}_{SP}(-h) = {} & [H(x + x_{SP}^0) - G(x - \max^m(x_{SP}^*, x_{SP}^0))] \\
& \times \int_0^{\tau_{SP1}} H\left(t - \tau - \frac{R}{\alpha} - (h + S(\tau)\sin\lambda\sin\delta)\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}\right) H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right) \mathbf{I}_{SP1} \, d\tau \\
& - [G(x - \max^m(x_{SP}^*, x_{SP}^0)) - H(x - x_{SP}^0)] \\
& \times \int_0^{\tau_{SP2}} H\left(t - \tau - \frac{R}{\alpha} - (h + S(\tau)\sin\lambda\sin\delta)\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}\right) H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right) \mathbf{I}_{SP1} \, d\tau \\
& - [G(x - \min^m(x_{SP}^{**}, x_{SP}^0)) - G(x - \min^m(x_{SP}^*, x_{SP}^0))] \\
& \times \int_{\tau_{SP3}}^{\tau_{SP2}} H\left(t - \tau - \frac{R}{\alpha} - (h + S(\tau)\sin\lambda\sin\delta)\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}\right) H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right) \mathbf{I}_{SP1} \, d\tau \\
& - \mathbf{u}_{S1} + \mathbf{u}_{S2} + [H(\zeta_1 + \zeta_{B1}^0) - G(\zeta_1 - \max^m(\zeta_{B1}^*, \zeta_{B1}^0))] \\
& \times \int_0^{\tau_{B1}} H\left(t - \tau - \frac{R}{\alpha} - (h + S(\tau)\sin\lambda\sin\delta)\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}\right) H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right) \mathbf{I}_{SP2} \, d\tau \\
& - [G(\zeta_1 - \max^m(\zeta_{B1}^*, \zeta_{B1}^0)) - H(\zeta_1 - \zeta_{B1}^0)] \\
& \times \int_0^{\tau_{B2}} H\left(t - \tau - \frac{R}{\alpha} - (h + S(\tau)\sin\lambda\sin\delta)\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}\right) H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right) \mathbf{I}_{SP2} \, d\tau \\
& - [G(\zeta_1 - \min^m(\zeta_{B1}^{**}, \zeta_{B1}^0)) - G(\zeta_1 - \min^m(\zeta_{B1}^*, \zeta_{B1}^0))] \\
& \times \int_{\tau_{B3}}^{\tau_{B2}} H\left(t - \tau - \frac{R}{\alpha} - (h + S(\tau)\sin\lambda\sin\delta)\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}\right) H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right) \mathbf{I}_{SP2} \, d\tau.
\end{aligned} \tag{29}$$

The fourth and fifth terms in (29) namely $\mathbf{u}_{S1}$ and $\mathbf{u}_{S2}$ denote the expressions on the r.h.s. of (27) with integrand $\mathbf{I}_S$ replaced by $\mathbf{I}_{SP1} H(R/\rho - \beta/\alpha)$ and $\mathbf{I}_{SP2} H(R/\rho - \beta/\alpha)$ respectively.

In (28) and (29) the following meaning is attached to the symbol:

$$\begin{aligned}
G(\zeta_1 - \max^m(\zeta_{j1}^*, \zeta_{j1}^0)) &= H(\zeta_1 - \zeta_{j1}^*) H(L_j) && \text{if } \zeta_{j1}^* = \max^m(\zeta_{j1}^*, \zeta_{j1}^0) \\
&= H(\zeta_1 - \zeta_{j1}^0) && \text{if } \zeta_{j1}^0 = \max^m(\zeta_{j1}^*, \zeta_{j1}^0) \\
& && \text{or } \zeta_{j1}^* \text{ does not exist.}
\end{aligned} \tag{30}$$

Similar meaning is given for $G(\zeta_1 - \min^m(\zeta_{j1}^*, \zeta_{j1}^0))$ etc. Note the additional factor $H(L_j)$

whenever the starred member (viz. ζ_{j1}^{*} above) is to be considered. Also we have

$$\zeta_{j1}^{0} = [V_j^2 t^2 - \zeta_2^2 - \zeta_3^2]^{1/2},$$
$$x_{SP}^{0} = \left[\left(\alpha t - h\left(\frac{\alpha^2}{\beta^2} - 1\right)^{1/2}\right)^2 - y^2\right]^{1/2} \tag{31}$$
$$\zeta_{B1}^{0} = \left[\frac{th}{\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}} - \zeta_2^2 - \zeta_3^2\right]^{1/2}$$

Meaning of different symbols in (28) and (29) are given in Appendix 2.

5. NATURE OF WAVE ARRIVALS

The part of the surface displacement as given by (28) includes, besides the contribution from body waves namely P, S and SH wave at $\zeta_1 = \pm\zeta_{j1}^{0} = \pm[V_j^2 t^2 - \zeta_2^2 - \zeta_3^2]^{1/2}$ for $j = P$, S or SH or equivalently at $t = (\zeta_1^2 + \zeta_2^2 + \zeta_3^2)^{1/2}/V_j = (x^2 + y^2 + h^2)^{1/2}/V_j$ the contribution from the conical waves at

$$\zeta_1 = \zeta_{j1}^{*} \text{ (or } t = t_j^{*}) \quad \text{and} \quad \zeta_1 = \zeta_{j1}^{**} \text{ (or } t = t_j^{**}),$$

(for $j_1 = P$, S or SH) at places where $L_j > 0$ and $\tau_j^c > 0$ respectively.

Similarly (29) shows that the head wave, SP, arrives at

$$x = \pm x_{SP}^{0} \quad \text{or} \quad t = \frac{(x^2 + y^2)^{1/2}}{\alpha} + h\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2},$$

while conical head waves arrive at $x = x_{SP}^{*}$ (or $t = t_{SP}^{*}$) and $x = x_{SP}^{**}$ (or $t = t_{SP}^{**}$) at places where $L_{SP} > 0$ and $\tau_{SP}^{C} > 0$ respectively. Head waves and associated conical waves arise due to the presence of surface and are absent in an infinite medium.

Alternatively from (22) and (23), conical P or S waves can be regarded as the envelope of elementary P or S waves as the source moves. Thus

$$\zeta_1 = S(\tau) + \frac{V_j^2(t-\tau)}{\dot{S}(\tau)}, \tag{32}$$
$$(\zeta_2^2 + \zeta_3^2)^{1/2} = V_j(t-\tau)\left[1 - \frac{V_j^2}{[\dot{S}(\tau)]^2}\right]^{1/2},$$

represents the conical P or S waves at time t for $j = P$ or S, in terms of the parameter τ. The representation (32) shows that the necessary condition for the existence of conical wave is that $\dot{S}(\tau) > V_j$. This implies that the source must move supersonically with respect to the longitudinal wave velocity of the medium in order to generate a conical P-wave. Similar remarks hold for conical shear wave when the source must move supersonically w.r.t. the transverse wave velocity. The regions of existence and arrival times of P, S and the associated conical waves can be obtained easily by the intersection, with the surface $z = -h$, of the wave fronts in an infinite medium (Fig. 3a, b) which can be easily constructed by drawing the envelopes of elementary P or S-waves as the source moves.

We note that $\zeta_1 = \zeta_{B1}^{0}$, $\zeta_1 = \zeta_{B1}^{*}$ and $\zeta_1 = \zeta_{B1}^{**}$ or equivalently $t = t_B^0$, $t = t_B^{*}$ and $t = t_B^{**}$ do not represent any wave arrivals. This will be evident if instead of source at a depth h, we consider dislocation starting from the free surface and find the displacement inside the medium. The corresponding time arrivals are obtained from the present case on simply replacing h by z. In that case it is easily seen that $t = t_B^0$, $t = t_B^{*}$ and $t = t_B^{**}$ do not represent any wave fronts since they do not satisfy the eikonal equation.

Phases expected at any station on the surface in case of non-uniformly moving sources differ considerably from uniformly moving sources. This can be easily seen from a look at the table below.

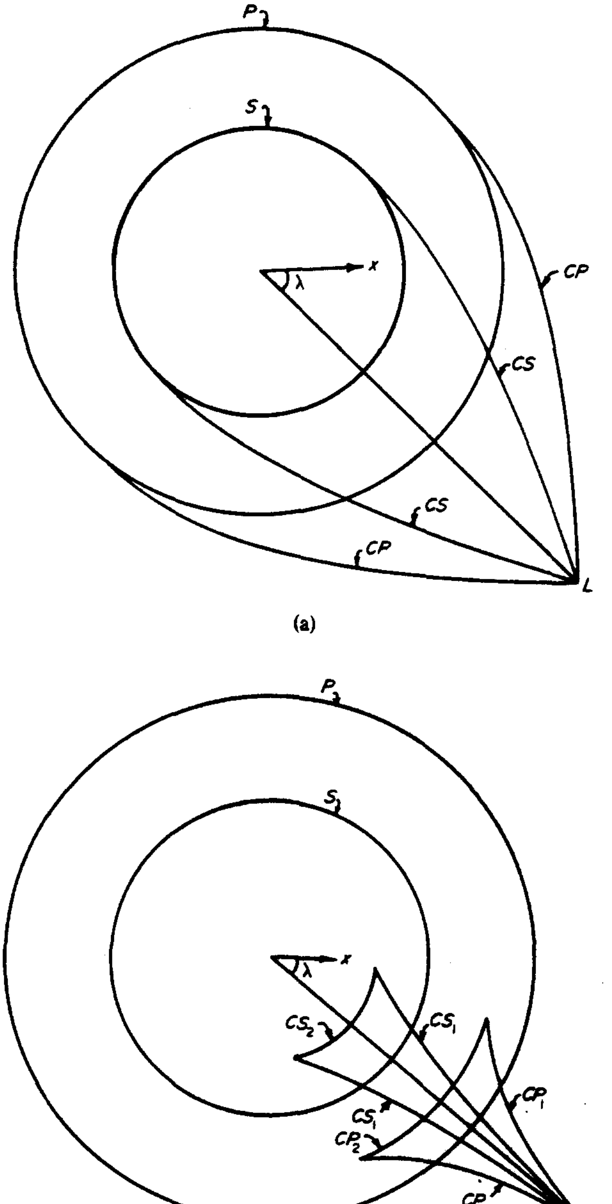

Fig. 3. Wave fronts in an infinite medium. (a) The case of a source L moving nonuniformly with starting velocity greater than the longitudinal velocity of the medium. P, S, CP and CS represent longitudinal, transverse, conical P and conical S wavefronts respectively. (b) The case of a rapidly accelerating source L starting with velocity less than the transverse wave velocity of the medium. P, S, CP_1, CP_2, CS_1, CS_2 represent longitudinal, transverse, first conical P, second conical P, first conical S and second conical S wave fronts respectively.

In Table 1 R_0/ρ_0 and R^*/ρ^* denote the values of R at $\tau = 0$ and $\tau = \tau^*_{\xi P}$ respectively. The condition for the existence of conical waves can be summarised as follows: for both an accelerating and decelerating source, if the initial velocity is greater than the representative velocity (i.e. longitudinal or transverse), first conical P, S or SP, wave always exist. Second conical P, S or SP waves do not exist in case of a decelerating source. They are always preceeded by first conical wave. They exist only in case of an accelerating source and when the initial velocity of source is less than the representative velocity (i.e. longitudinal or transverse as the case may be) and the acceleration is rapid enough.

In particular for a vertical fault plane (i.e. $\delta = \pi/2$) with the source moving with a constant velocity V at an angle λ to the strike direction, the arrival times and region of existence of

Table 1. Phases expected at the surface in case of moving source

Supersonic Source Non-uniform motion	Uniform motion	Subsonic source	Region of existence
First conical P	Conical P		$L_P > 0$
P	P	P	All station
Second conical P			$\tau_P{}^C > 0$
First conical SP	Conical SP		$\frac{R_0}{\rho_0} > \frac{\beta}{\alpha}$ and $L_{SP} > 0$
SP	SP	SP	$\frac{R_0}{\rho_0} > \frac{\beta}{\alpha}$
First conical S	Conical S		$L_S > 0$
S	S	S	All station
Second conical S			$\tau_S{}^C > 0$
Second conical SP			$\frac{R^*}{\rho^*} > \frac{\beta}{\alpha}$ and $\tau_{SP}^C > 0$
Rayleigh	Rayleigh	Rayleigh	All station

conical waves as obtained from (28) and (29) are found to agree with those of Roy[23], derived in a different way.

6. FIRST MOTIONS

The expressions for the surface displacements as given by (28) and (29) are in the form of double integrals over finite ranges. As such, computation of theoretical seismograms for any dislocation model can only be done with the help of a computer. However some idea about the nature of surface displacement can be made if one computes the first motion approximations to the surface displacement near different wave arrivals which can be easily done by a limiting process used earlier by Roy[22] in case of finite sources.

The surface displacement near P or S-wave arrivals is obtained, from (28) as

$$\mathbf{u}_j(-h) = \operatorname*{Lt}_{t\to(\rho_0/V_j)+0} \int_0^{\tau_{j1}} \mathbf{I}_j H\left(t-\tau-\frac{\rho}{V_j}\right) H(\zeta_1+\zeta_{j1}^0)\,d\tau, \tag{33}$$

where

$$\rho_0 = (x^2+y^2+h^2)^{1/2}.$$

We first consider the case when the source is moving subsonically (i.e. initial velocity less than S-wave velocity) then the surface displacement near P- or S-wave arrivals, as obtained from (33), is

$$\mathbf{u}_j(-h) \simeq \operatorname*{Lt}_{\substack{t\to(\rho_0/V_j)+0\\ \tau_{j1}\to 0+}} \tau_{j1}\{\mathbf{I}_j\} H\left(t-\frac{\rho_0}{V_j}\right), \tag{34}$$

where

$$\begin{aligned} \{\mathbf{I}_j\} &= \text{jump of } \mathbf{I}_j \text{ at } t = \psi_j(\tau) = \tau + \frac{\rho}{V_j} \\ &= \frac{\alpha}{2\pi\rho}\mathbf{M}_j(q_{jr}, 0)\frac{h + S(\tau)\sin\lambda\sin\delta}{\rho}, \\ q_{jr} &= \frac{i\alpha(t-\tau)R}{\rho^2}, \end{aligned} \tag{35}$$

and τ_{j1} is the root of $t-\psi_j(\tau) = t-\tau-\rho/V_j = 0$.

We consider the case when the source starts with finite velocity. In this case, let $S(\tau)$ have a Taylor's expansion near $\tau = 0$. Then we have

$$t-\tau-\frac{\rho}{V_j} = t-\frac{\rho_0}{V_j}-\tau\left[1-\frac{\zeta_1\dot{S}(0)}{\rho_0 V_j}\right]. \tag{36}$$

Then for sources moving initially with subsonic speed, the displacement near P or S wave arrivals is obtained from (33)–(35) as

$$\mathbf{u}_j(-h) \simeq \frac{\left(t-\frac{\rho_0}{V_j}\right)}{\left[1-\frac{\zeta_1 \dot{S}(0)}{\rho_0 V_j}\right]} \frac{\alpha h \mathbf{M}_j(q_{j0}, 0)}{2\pi\rho_0^2} H\left(t-\frac{\rho_0}{V_j}\right) H\left(1-\frac{\zeta_1 \dot{S}(0)}{\rho_0 V_j}\right) \tag{37}$$

where

$$q_{j0} = q_{j\tau}\big|_{\tau=0} = \frac{i(x^2+y^2)^{1/2}}{\rho_0} = \frac{iR_0}{\rho_0}. \tag{38}$$

In case the source is moving supersonically i.e. if the initial velocity is greater than P-wave velocity, then for $(\zeta_1 \dot{S}(0)/(\rho_0 V_j)) < 1$, assuming again $S(\tau)$ has a Taylor's expansion near $\tau = 0$ the surface displacement near P- or S-wave arrivals is again given by (37).

If $S(\tau)$ has no Taylor's expansion near $\tau = 0$, then let $S(\tau)$ has the following form near $\tau = 0$, namely

$$S(\tau) = S_0 \tau^n, \quad S_0 = \text{constant}(>0), \tag{39}$$

If $n > 1$, then (37) gives the surface displacement near $\tau = \rho_0/V_j$ after setting $\dot{S}(0) = 0$ in (37). Let $n < 1$, in this case initial velocity is infinite, i.e. $\dot{S}(0) \to \infty$ as $t \to 0+$ and we have

$$t - \tau - \frac{\rho}{V_j} = t - \frac{\rho_0}{V_j} + \frac{S_0 \zeta_1}{\rho_0 V_j} \tau^n. \tag{40}$$

In this case for $\zeta_1 < 0$, we get the surface displacement near $t = \rho_0/V_j$ from (33) as

$$\mathbf{u}_j(-h) \simeq \frac{\left(t-\frac{\rho_0}{V_j}\right)^{1/n}}{\left(-\frac{S_0 \zeta_1}{\rho_0 V_j}\right)^{1/n}} \frac{\alpha h \mathbf{M}_j(q_{j0}, 0)}{2\pi\rho_0^2} H\left(t-\frac{\rho_0}{V_j}\right) H(-\zeta_1). \tag{41}$$

In case the source is moving initially with supersonic velocity then for $\dot{S}(0)\zeta_1/(\rho_0 V_j) > 1$ or if the source is moving with infinite velocity initially namely when $S(\tau) = S_0\tau^n$, $n < 1$ then for $\zeta_1 > 0$ i.e. at places where conical P- or S-wave exists, then surface displacements associated with P- or S-wave are obtained from (28) as

$$\mathbf{u}_j(-h) = \int_{\tau_{j2}}^{\tau_{j1}} \mathbf{I}_j H\left(t - \tau - \frac{\rho}{V_j}\right) d\tau.$$

In this case the surface displacement near $t = \rho_0/V_j$ cannot be obtained by the limiting process described above.

The surface displacement near first conical P- or S-wave arrival, as obtained from third terms in (28) and (29), is in the form

$$\begin{aligned} \mathbf{u}_j(-h) &\simeq \operatorname*{Lt}_{t \to t_j^*+} \int_{\tau_j^*}^{\tau_j^*+\epsilon} \mathbf{I}_j H(L_j) H(t - \psi_j(\tau))\, d\tau \\ &\simeq \operatorname*{Lt}_{\substack{t \to t_j^*+ \\ \epsilon \to 0+}} \epsilon \{\mathbf{I}_j\}_{\tau=\tau_j^*} H(t - \psi_j(\tau_j^*)) H(L_j) \end{aligned} \tag{42}$$

where $\{\mathbf{I}_j\}$ is given by (35) and ϵ is obtained from the relation that $\tau_j^* + \epsilon$ is the root of $t - \psi_j(\tau) = t - \tau - \rho/V_j = 0$. Now

$$t - \psi_j(\tau_j^* + \epsilon) = t - t_j^* - \frac{\epsilon^2}{2} \psi''(\tau_j^*) \tag{43}$$

since $\psi'(\tau_j^*) = 0$ and $t_j^* = \psi_j(\tau_j^*)$.

Hence from (42) and (43) we get as $t \to t_j^* +$

$$\mathbf{u}_j(-h) \simeq \frac{2^{1/2}(t - t_j^*)^{1/2}}{[\psi''(\tau_j^*)]^{1/2}} \{\mathbf{I}_j\}_{\tau=\tau_j^*} H(t - t_j^*) H(L_j). \tag{44}$$

Similarly the surface displacement near the arrival time of second conical P- or S-wave is given by

$$\mathbf{u}_j(-h) \simeq \frac{2^{1/2}(t - t_j^{**})^{1/2}}{[\psi''(\tau_j^{**})]^{1/2}} \{\mathbf{I}_j\}_{\tau=\tau_j^{**}} H(t - t_j^{**}) H(\tau_j^C). \tag{45}$$

Near the cusp of $t - \psi_j(\tau) = t - \tau - \rho/V_j = 0$, where the two extremum $\tau = \tau_j^*$ and $\tau = \tau_j^{**}$ coalesce, with the common value of t and τ being denoted by t_j^C, τ_j^C respectively, the surface displacements can be written as

$$\mathbf{u}_j(-h) \simeq \frac{6^{1/3}(t - t_j^C)^{1/3}}{[\psi'''(\tau_j^C)]^{1/3}} \{\mathbf{I}_j\}_{\tau=\tau_j^C} H(t - t_j^C) H(\tau_j^C). \tag{46}$$

In case of the head wave, the surface displacement near its arrival time is given by

$$\mathbf{u}_{SP}(-h) \simeq \operatorname*{Lt}_{t \to t_{SP}+} \int_0^{\tau_{SP1}} \mathbf{I}_S H(t - \psi_{SP}(\tau)) H\left(\frac{R}{\rho} - \frac{\beta}{\alpha}\right) \mathrm{d}\tau, \tag{47}$$

where

$$t_{SP} = \psi_{SP}(0) = \frac{R}{\alpha} + h\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2},$$

$$\psi_{SP}(\tau) = \tau + \frac{R}{\alpha} + (h + S(\tau) \sin\lambda \sin\delta)\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}, \tag{48}$$

and τ_{SP1} is the root of $t - \psi_{SP}(\tau) = 0$.

Following Roy [22], (47) can be written as

$$\mathbf{u}_{SP}(-h) \simeq (\mathbf{G}_{SP})_{\tau=0} \operatorname*{Lt}_{\substack{t \to t_{SP}+0 \\ \tau_{SP1} \to 0+}} \int_0^{\tau_{SP1}} (t - t_{SP} - \psi_{SP}(\tau) + t_{SP})\,\mathrm{d}\tau \; H(t - t_{SP}) H\left(\frac{R_0}{\rho_0} - \frac{\beta}{\alpha}\right) \tag{49}$$

where

$$G_{SP} = -\frac{\alpha^2}{\pi R^{1/2}} \frac{Re(\mathbf{E}_{SP})\left(\frac{\alpha^2}{\beta^2} - 1\right)^{3/4}}{\left[R\left(\frac{\alpha^2}{\beta^2} - 1\right)^{1/2} - Z\right]^{3/2}},$$

$$\mathbf{E}_{SP} = \operatorname*{Lt}_{\substack{q_{SP1} \to i \\ \omega_{SP1} \to 0}} \frac{\mathbf{M}_S(q_{SP1}, \omega_{SP1})}{(q_{SP1}^2 + \omega_{SP1}^2 + 1)^{1/2}}. \tag{50}$$

Exactly similar to the previous case we get

$$\mathbf{u}_{SP}(-h) \simeq \frac{1}{2}(\mathbf{G}_{SP})_{\tau=0} \frac{(t - t_{SP})^2}{[\psi_{SP}'(0)]} H(t - t_{SP}) H(\psi_{SP}'(0)) H\left(\frac{R_0}{\rho_0} - \frac{\beta}{\alpha_0}\right), \tag{51}$$

when $S(\tau)$ has a Taylor's expansion near $\tau = 0$ and

$$\mathbf{u}_{SP}(-h) \simeq (\mathbf{G}_{SP})_{\tau=0} \frac{(t - t_{SP})^{(n+1)/n}}{(S_0 L)^{1/n}} \left(\frac{n}{n+1}\right) H(L) H(t - t_{SP}) H\left(\frac{R_0}{\rho_0} - \frac{\beta}{\alpha}\right), \tag{52}$$

when $S(\tau) = S_0 \tau^n$ near $\tau = 0$ and

$$L = -\frac{x\cos\lambda + y\sin\lambda\cos\delta}{\alpha(x^2 + y^2)^{1/2}} + \sin\lambda\sin\delta\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}.$$

Exactly in a similar way the surface displacement near the arrival times of first conical head wave is given as

$$\mathbf{u}_{SP}(-h) \simeq \frac{2^{3/2}}{3} \frac{(t - t^*_{SP})^{3/2}}{[\psi''_{SP}(\tau^*_{SP})]^{1/2}} (\mathbf{G}_{SP})_{\tau=\tau^*_{SP}} H(L_{SP}) H(t - t^*_{SP}) H\left(\frac{R^*}{\rho^*} - \frac{\beta}{\alpha}\right). \tag{53}$$

The corresponding result near the arrival times of second conical head wave is obtained from (53) on replacing t^*_{SP}, τ^*_{SP} and $H(L)$ by t^{**}_{SP}, τ^*_{SP} and $H(\tau^C_{SP})$ respectively.

At places where the two conical head waves coalesce, the first motion approximation to the surface displacement is given by

$$\mathbf{u}_{SP}(-h) \simeq \frac{3}{4} \frac{6^{1/3}(t - t^C_{SP})^{4/3}}{[\psi'''(\tau^C_{SP})]^{1/3}} (\mathbf{G}_{SP})_{\tau^C_{SP}} H(\tau^C_{SP}) H(t - t^C_{SP}), \tag{54}$$

where (t^C_{SP}, τ^C_{SP}) are the common values of (t^*_{SP}, t^{**}_{SP}) and $(\tau^*_{SP}, \tau^{**}_{SP})$ respectively at the cusp of $t - \psi_{SP}(\tau) = 0$ and $\psi'''(\tau_{SP}) = (d^3/d\tau^3)\psi(\tau)$.

7. DISCUSSION

It is seen from Sections 5 and 6 that the surface displacements in case of a nonuniformly moving source differ from those in case of a stationary source both in number of wave arrivals and amplitude of different phases. Thus if $\mathbf{u}_j$ ($j = P$, S or SH) be the surface displacement associated with first motion of the P, SV or SH wave arrivals in case of a nonuniformly expanding dislocation and $[\mathbf{u}_j]_0$ be the corresponding quantities in case of a stationary source, then from (37)

$$\mathbf{u}_j = M_j[\mathbf{u}_j]_0,$$

where, for $j = P$, S or SH

$$M_j = \frac{1}{\left[1 - \dfrac{\zeta_1 \dot{S}(0)}{\rho_0 V_j}\right]} = \frac{1}{1 - \dfrac{\dot{S}(0)\cos\phi}{V_j}},$$

where ϕ is the angle made by the radius vector joining the observation point and initial position of the source with the direction of propagation.

Thus the effect of motion of the source is to multiply first motion responses by a scalar factor M_j which depends only on the initial velocity of the source. A similar factor M_j was also obtained by Savage[13] in connection with a source moving with uniform velocity. Since the modulation factor M_j is positive for subsonic source and the nodal plane determination depends only on the sign of initial P motion, the motion of the source does not alter the nodal position. As the first motion responses are uneffected by the acceleration or deceleration of the source, to analyse the nature of motion one needs to study the amplitude of body waves after the first arrivals.

In case of a source moving initially with infinite velocity, a change of pulse shape is expected near body wave arrivals. Thus for a source moving initially with infinite velocity as seen from Section 6, the surface displacement near P, SV or SH wave arrivals varies as $(t - \rho_0/V_j)^{1/n} H(t - \rho_0/V_j)$ while in case of a source starting with finite velocity it varies as $(t - \rho_0/V_j) H(t - \rho_0/V_j)$. In case of a source moving supersonically (i.e. $\dot{S}(0) > V_j$) conical body phases appear and near their arrival time the surface displacement behaves as $(t - t^*_j)^{1/2} H(t - t^*_j)$ where t^*_j, is the time of arrival of conical phase.

We have presented a method for obtaining theoretical seismograms for a particular earthquake source model, viz. gliding edge dislocation, in which finiteness and nonuniform motion of the source have been taken into account. As eqn (7) is valid for any arbitrary body force system, the present analysis can be used to obtain the surface displacement in an elastic half space for other realistic earthquake source model, viz. skew dislocation, etc. incorporating the effect of actual motion of the source. Recently two dimensional dislocation models have been used to explain accelerograph data of different earthquakes[9, 10, 15, 16]. In case of two dimensional dislocations the surface displacements will be expressed in terms of a single integral instead of double integrals as in the case of three dimensional ones. It is hoped to compute theoretical seismogram in case of two dimensional dislocation source models in some future publications. Comparison of theoretical seismogram, following the present analyses with phases of an actual earthquake may give a better evaluation of the relevant parameters, viz. length, rupture velocity etc. associated with faulting.

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APPENDIX 1

Meaning of different symbols used in (14) and (16):

$$\mathbf{M}_P(q,\omega)=\frac{A_P}{k_P\nu_P F(\xi,\eta)}[-2\zeta_S(\xi\mathbf{e}_x+\eta\mathbf{e}_y)+i(2\xi^2+2\eta^2+\alpha^2/\beta^2)\mathbf{e}_z],$$

$$\mathbf{M}_S(q,\omega)=\frac{A_S}{k_S\nu_S F(\xi,\eta)}[(2\xi^2+2\eta^2+\alpha^2/\beta^2)(\xi\mathbf{e}_x+\eta\mathbf{e}_y)-2i(\xi^2+\eta^2)\zeta_P\mathbf{e}_z],$$

$$\mathbf{M}_{SH}(q,\omega)=\frac{\alpha^2 A_{SH}}{\beta^2 k_S\nu_S(\xi^2+\eta^2)}(\eta\mathbf{e}_x-\xi\mathbf{e}_y),$$

$$F(\xi,\eta)=(2\xi^2+2\eta^2+\alpha^2/\beta^2)^2-4(\xi^2+\eta^2)\zeta_P\zeta_S,$$

$$\xi=(q\cos\phi-\omega\sin\phi),\qquad \eta=q\sin\phi+\omega\cos\phi,$$

$$\zeta_P=(q^2+\omega^2+1)^{1/2},\qquad \zeta_S=(q^2+\omega^2+\alpha^2/\beta^2)^{1/2},$$

$$A_P=-2i\xi\eta\cos\lambda\sin\delta+2\xi\zeta_P\cos\lambda\cos\delta+2\eta\zeta_S\sin\lambda\cos 2\delta-i(\eta^2+\zeta_P^2)\sin\lambda\sin 2\delta,$$

$$A_S(\xi^2+\eta^2)=-2i\xi\eta\zeta_S\cos\lambda\sin\delta+\xi(2\xi^2+2\eta^2+\alpha^2/\beta^2)\cos\lambda\cos\delta$$
$$+\eta(2\xi^2+2\eta^2+\alpha^2/\beta^2)\sin\lambda\cos 2\delta-i\zeta_S(2\eta^2+\xi^2)\sin\lambda\sin 2\delta$$

$$A_{SH}=-i(\eta^2-\xi^2)\cos\lambda\sin\delta+\eta\zeta_S\cos\lambda\cos\delta+i\xi\eta\sin\lambda\sin 2\delta-\xi\zeta_S\sin\lambda\cos 2\delta,$$

$$k_P=\xi\cos\lambda+\eta\sin\lambda\cos\delta-i\zeta_P\sin\lambda\sin\delta,$$

$$k_S=\xi\cos\lambda+\eta\sin\lambda\cos\delta-i\zeta_S\sin\lambda\sin\delta,$$

$$\nu_P=-\xi\sin\lambda+\eta\cos\lambda\cos\delta-i\zeta_P\cos\lambda\sin\delta,$$

$$\nu_S=-\xi\sin\lambda+\eta\cos\lambda\cos\delta-i\zeta_S\cos\lambda\sin\delta.$$

Note that ξ, η as defined above is the same as ξ, η as given by (11) except for a factor p/α.

The other symbols in (15) and (17) are as follows:

$$q_P=\frac{\alpha}{\rho^2}[i(t-\tau)R+Z((t-\tau)^2-\rho^2/\alpha^2)^{1/2}\cos\psi],$$

$$\omega_P=\left[\frac{\alpha^2(t-\tau)^2}{\rho^2}-1\right]^{1/2}\sin\psi,$$

$$R=[(x-S(\tau)\cos\lambda)^2+(y-S(\tau)\sin\lambda\cos\delta)^2]^{1/2},$$

$$Z=(h+S(\tau)\sin\lambda\sin\delta),\qquad \rho=(R^2+Z^2)^{1/2},$$

$$q_S = \frac{\alpha}{\rho^2}[i(t-\tau)R + Z((t-\tau)^2 - \rho^2/\beta^2)^{1/2}\cos\psi],$$

$$\omega_S = \frac{\alpha}{\beta}\left[\frac{\beta^2(t-\tau)^2}{\rho^2} - 1\right]^{1/2},$$

$$q_{SP1} = \frac{i\alpha}{\rho^2}\left[(t-\tau)R - \frac{Z\rho^2}{\beta R}(m^2\chi^2 + m^2\sin^2\psi)^{1/2}\right],$$

$$\omega_{SP1} = \left[\left(\frac{\alpha(t-\psi_{SP}(\tau))}{R} + 1\right)^2 - 1\right]^{1/2}\sin\psi,$$

$$\psi_{SP}(\tau) = \tau + \frac{R}{\alpha} + Z\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2},$$

$$m = (\delta'^2 - m^2\chi^2)^{1/2},$$

$$m\chi = \frac{R}{\rho}\left[1 - \frac{\beta^2(t-\tau)^2}{\rho^2}\right]^{1/2},$$

$$\delta' = \left(1 - \frac{\beta^2}{\alpha^2}\right)^{1/2} - \frac{\beta(t-\tau)Z}{\rho^2},$$

$$q_{SP2} = \frac{i\alpha(t-\tau)R}{\rho^2} - i\frac{\alpha}{\beta}\frac{Z}{R}\delta'\sin\psi,$$

$$\omega_{SP2} = \frac{\alpha^2}{\beta^2}\left(\frac{\beta^2(t-\tau)^2}{\rho^2} - 1\right) + \left[\left(\frac{\alpha(t-\psi_{SP}(\tau))}{R} + 1\right)^2 - 1 - \frac{\alpha^2}{\beta^2}\left(\frac{\beta^2(t-\tau)^2}{\rho^2} - 1\right)\right]\sin^2\psi.$$

APPENDIX 2

In (28) L_j ($j = P$, S or SH) has following values, depending on the nature of moving source;

(a) $-1 + \zeta_1\dot{S}(0)/(V_j\rho_0)$ if $S(\tau)$ has Taylor's expansion near $\tau = 0$ and $\dot{S}(0) > V_j$.

(b) ζ_1, if $\dot{S}(0)$ is infinite and for a decelerating source.

(c) τ_j^C, when both ζ_{j1}^* and ζ_{j1}^{**} exist; (i.e. in case of an accelerating source with initial velocity either equal to zero or less than P or S velocity depending on having the value P or S) and τ_j^C denotes the common value of (τ_j^*, τ_j^{**}).

The corresponding values of L_{SP} are:

(a) $$-1 + \dot{S}(0)\left[\frac{x\cos\lambda + y\sin\lambda\cos\delta}{\alpha(x^2+y^2)^{1/2}}\right] - \sin\lambda\sin\delta\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}$$

(b) $$-\sin\lambda\sin\delta\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2} + \frac{x\cos\lambda + y\sin\lambda\cos\delta}{\alpha(x^2+y^2)^{1/2}}$$

(c) $$\tau_{SP}^C.$$

Also the values of L_B are (a) τ_B^* if $\zeta_B^*{}^\dagger$ exists and (b) $\tau_B{}^C$ if both ζ_{B1}^* and $\zeta_B^*{}^\dagger$ exist.

τ_j^C, τ_{SP}^C and $\tau_B{}^C$ denote the values of τ where the function

$$\psi_j(\tau) = \tau + \frac{\rho}{V_j}, \qquad \psi_B(\tau) = \tau + \frac{\rho^2}{Z}\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}$$

$$\psi_{SP}(\tau) = \tau + \frac{R}{\alpha} + Z\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)^{1/2}$$

have vanishing first and second derivatives respectively.